

$$\dot{z}_1 = \sigma_1 z_1 + \delta_1 \bar{z}_2 \bar{z}_3 + \beta_1 \bar{z}_1 z_4 + [\kappa_{11} |z_1|^2 + \kappa_{12} (|z_2|^2 + |z_3|^2)] z_1 + [\mu_{11} |z_4|^2 + \mu_{12} (|z_5|^2 + |z_6|^2)] z_1 + \nu_1 \bar{z}_1 \bar{z}_5 \bar{z}_6 + \xi_1 z_2 z_3 z_4 + \eta_1 (\bar{z}_2 z_3 \bar{z}_6 + z_2 \bar{z}_3 \bar{z}_5),$$

$$\dot{z}_4 = \sigma_2 z_4 + \delta_2 \bar{z}_5 \bar{z}_6 + \beta_2 z_1^2 + [\kappa_{21} |z_1|^2 + \kappa_{22} (|z_2|^2 + |z_3|^2)] z_4 + [\mu_{21} |z_4|^2 + \mu_{22} (|z_5|^2 + |z_6|^2)] z_4 + \nu_2 z_1 \bar{z}_2 \bar{z}_3 + \xi_2 (\bar{z}_3^2 \bar{z}_5 + \bar{z}_2^2 \bar{z}_6).$$

$$I_1 = \mathbb{C}(0, 0, 0, 1, 0, 0)$$

$$I_2 = \mathbb{C}\{(1, 0, 0, 0, 0, 0), (0, 0, 0, 1, 0, 0)\}$$

$$I_3 = \mathbb{C}\{(0, 0, 0, 1, 0, 0), (0, 0, 0, 0, 1, 0), (0, 0, 0, 0, 0, 1)\}$$

$$I_4 = \mathbb{C}\{(1, 0, 0, 0, 0, 0), (0, 0, 0, 1, 0, 0), (0, 0, 0, 0, 1, 0), (0, 0, 0, 0, 0, 1)\}$$

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## 正六角形格子上でのパターン形成

$$I_3 = \mathbb{C}\{(0, 0, 0, 1, 0, 0), (0, 0, 0, 0, 1, 0), (0, 0, 0, 0, 0, 1)\}$$

Busse, F. H. 1967 The stability of finite amplitude cellular convection and its relation to an extremum principle. *J. Fluid Mech.* **30**, 625–649. (doi:10.1017/S0022112067001661)

Buzano, E. & Golubitsky, M. 1983 Bifurcation on the hexagonal lattice and the planar Bénard problem. *Phil. Trans. R. Soc. A* **308**, 617–667. (doi:10.1098/rsta.1983.0018)

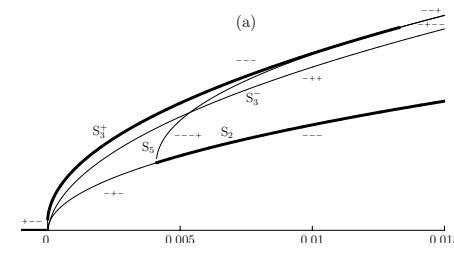
Golubitsky, M., Swift, J. W. & Knobloch, E. 1984 Symmetries and pattern selection in Rayleigh–Bénard convection. *Physica D* **10**, 249–276. (doi:10.1016/0167-2789(84)90179-9)

Fujimura & Yamada 2008 *Proc. R. Soc. Ser. A* **464**, 2721–273.

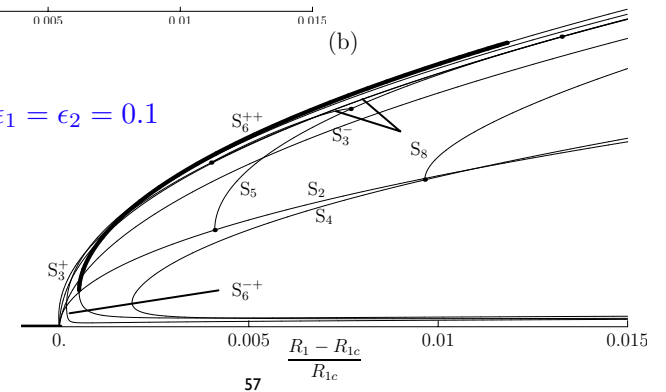
56

55

$$I_3 = \mathbb{C}\{(0, 0, 0, 1, 0, 0), (0, 0, 0, 0, 1, 0), (0, 0, 0, 0, 0, 1)\}$$

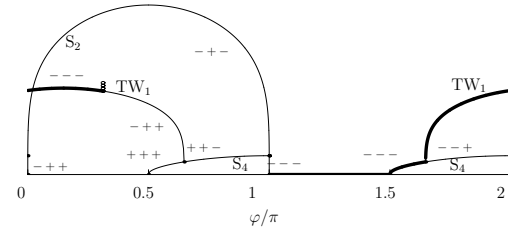


$$P_1 = P_2 = 7, \quad \epsilon_1 = \epsilon_2 = 0.1$$

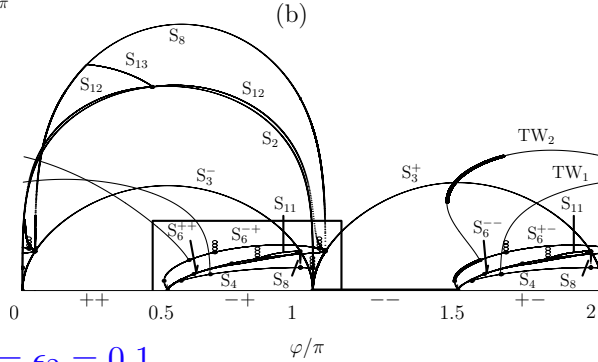


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$$I_2 = \mathbf{C}\{(1, 0, 0, 0, 0, 0), (0, 0, 0, 1, 0, 0)\} \quad \sigma_1 = \rho \cos \varphi, \quad \sigma_2 = \rho \sin \varphi$$



$$\rho = 10^{-4}$$



$$P_1 = P_2 = 7, \quad \epsilon_1 = \epsilon_2 = 0.1$$

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その代わりとして,

$$\xi = \frac{2t}{T} - 1 \quad \leftrightarrow \quad t \in [0, T] \quad \text{and} \quad \xi \in [-1, 1].$$

とおく, すると次を得る:

$$\tau \frac{d\mathbf{x}}{d\xi} = \mathbf{f}(\mathbf{x}, \mu) \quad \text{where} \quad \tau = \frac{2}{T}$$

擬弧長:

$$(\delta s)^2 = (\delta x)^2 + (\delta \mu)^2 + (\delta \tau)^2$$

$$\mathbf{x} = \bar{\mathbf{x}} + \hat{\mathbf{x}}, \quad \mu = \bar{\mu} + \hat{\mu}, \quad \text{and} \quad \tau = \bar{\tau} + \hat{\tau}$$

⇒

$$(\bar{\tau} + \hat{\tau}) \frac{d(\bar{\mathbf{x}} + \hat{\mathbf{x}})}{d\xi} = \mathbf{f}(\bar{\mathbf{x}} + \hat{\mathbf{x}}, \bar{\mu} + \hat{\mu}) \quad \text{and} \quad \bar{\mathbf{x}}(-1) + \hat{\mathbf{x}}(-1) = \bar{\mathbf{x}}(1) + \hat{\mathbf{x}}(1).$$

$$(s_n - s_{n-1})^2 = (\bar{\mathbf{x}} + \hat{\mathbf{x}} - x_{n-1})^2 + (\bar{\mu} + \hat{\mu} - \mu_{n-1})^2 + (\bar{\tau} + \hat{\tau} - \tau_{n-1})^2$$

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## 周期軌道に対するニュートン反復法

次の問題を考える:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mu), \quad \mathbf{x} \in \mathbb{R}^n, \quad \mu \in \mathbb{R}, \quad \mathbf{f}: \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^n$$

$\mathbf{x}(t) = \mathbf{x}(t+T)$  for  $T \gg 1$  であると仮定し, つぎのようにおく:

$$\xi = \frac{t}{T} \quad \leftrightarrow \quad t \in [0, T] \quad \text{and} \quad \xi \in [0, 1]$$

$$\tau \frac{d\mathbf{x}}{d\xi} = \mathbf{f}(\mathbf{x}, \mu) \quad \text{where} \quad \tau = \frac{1}{T}$$

$$\mathbf{x}(\xi) = \phi_\xi(\mathbf{x}(0), 0; \tau, \mu)$$

与えられたパラメター  $\mu$  に対し次を解く:

$$\mathbf{x}(0) = \phi_1(\mathbf{x}(0), 0; \tau, \mu) \quad \text{for} \quad \mathbf{x}(0) \quad \text{and} \quad \tau$$

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線形化すると,

$$\bar{\tau} \frac{d\hat{\mathbf{x}}}{d\xi} + \hat{\tau} \frac{d\bar{\mathbf{x}}}{d\xi} - \frac{d\mathbf{f}}{d\mathbf{x}}(\bar{\mathbf{x}}, \bar{\mu})\hat{\mathbf{x}} - \frac{\partial \mathbf{f}}{\partial \mu}(\bar{\mathbf{x}}, \bar{\mu})\hat{\mu} = -\bar{\tau} \frac{d\bar{\mathbf{x}}}{d\xi} + \mathbf{f}(\bar{\mathbf{x}}, \bar{\mu}) + O(2)$$

$$2(\bar{\mathbf{x}} - x_{n-1})\hat{\mathbf{x}} + 2(\bar{\mu} - \mu_{n-1})\hat{\mu} + 2(\bar{\tau} - \tau_{n-1})\hat{\tau}$$

$$= \delta s^2 - (\bar{\mathbf{x}} - x_{n-1})^2 - (\bar{\mu} - \mu_{n-1})^2 - (\bar{\tau} - \tau_{n-1})^2$$

$$\text{and} \quad \hat{\mathbf{x}}(-1) - \hat{\mathbf{x}}(1) = -\bar{\mathbf{x}}(-1) + \bar{\mathbf{x}}(1).$$

$\bar{\mathbf{x}}(\xi)$  と  $\hat{\mathbf{x}}(\xi)$  を次のようにチェビシェフ多項式に展開する:

$$\bar{\mathbf{x}}_k(\xi) = \sum_j A_j^{(k)} \tilde{T}_j(\xi) \quad \text{and} \quad \hat{\mathbf{x}}_k(\xi) = \sum_j \hat{a}_j^{(k)} \tilde{T}_j(\xi)$$

さらに, 任意の位相シフトが生じないように, 適当な  $l$  に対して

$$x_l(-1) = x_l(1) = 0$$

を課す.

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## Traveling Waves TW<sub>2</sub>

$$z_j(t) = r_j(t) e^{i\theta_j(t)}, \quad j = 1, \dots, 6, \quad \Phi_1 = \theta_1 + \theta_2 + \theta_3, \quad \Phi_2 = \theta_4 + \theta_5 + \theta_6,$$

$$\Theta_1 = \theta_4 - 2\theta_1, \quad \Theta_2 = \theta_5 - 2\theta_2, \quad \Theta_3 = \theta_6 - 2\theta_3$$

We require  $r_2 = r_3, r_5 = r_6, \Phi_1, \Phi_2, \Theta_1, \Theta_2, \Theta_3 : \text{const.}$

$\dot{\theta}_j \Rightarrow \text{const.} \Rightarrow \theta_j(t) = \tilde{\theta}_j t + \vartheta_j, \quad (j = 1, \dots, 6)$  for const.  $\tilde{\theta}_j, \vartheta_j$ .

Setting  $\tilde{\theta}_1/k = c$  and  $\xi = x - ct$ , we have

$$\begin{aligned} \hat{\psi} = & r_1 \phi_1 e^{ik\xi + i\vartheta_1} + r_2 \phi_1 e^{ik(-\xi/2 + \sqrt{3}y/2) + i\vartheta_2} + r_2 \phi_1 e^{ik(-\xi/2 - \sqrt{3}y/2) + i\vartheta_3} \\ & + r_4 \phi_4 e^{2ik\xi + i(\vartheta_1 + \Theta_1)} + r_5 \phi_4 e^{ik(-\xi + \sqrt{3}y) + i(2\vartheta_2 + \Theta_2)} + r_5 \phi_4 e^{ik(-\xi - \sqrt{3}y) + i(2\vartheta_3 + \Theta_2)} \\ & + c.c. + \text{higher order terms.} \end{aligned}$$

TW<sub>2</sub> lie on the group orbit  $\gamma z$  for  $\gamma = (\tilde{\theta}_1 t, -\tilde{\theta}_1 t/2) \in T^2$ .

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## Traveling Waves TW<sub>2</sub>



Proc. R. Soc. A Vol. 465 (2009) の1月号から12月号までの表紙の図案として採用された。

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## Oscillatory Solution in $\mathbb{C}^6$

We require  $r_1 = r_2 = r_3, r_4 = r_5 = r_6, \Theta_1 = \Theta_2 = \Theta_3 = 0$ , and  $\Phi_1 = 0$

$$\Rightarrow \dot{\theta}_j = 0$$

The equations governing the oscillatory solution OS: two-dimensional for  $r_1, r_4$

$$\dot{r}_1 = [\sigma_1 + \beta_1 r_4 + \delta_1 r_1 + (\kappa_{11} + 2\kappa_{12})r_1^2 + (\mu_{11} + 2\mu_{12} + \nu_1)r_4^2 + (2\eta_1 + \xi_1)r_1 r_4]r_1,$$

$$\dot{r}_4 = \sigma_2 r_4 + \beta_2 r_1^2 + \delta_2 r_4^2 + (\kappa_{21} + 2\kappa_{22} + 2\xi_2)r_1^2 r_4 + (\mu_{21} + 2\mu_{22})r_4^3 + \nu_2 r_1^3.$$

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## O(2) の下での近ヘテロクリニック軌道の周期

Higher harmonic resonances in vertical slot

Fujimura Phil.Trans.R.Soc. 340 (1992) pp.95 - 130

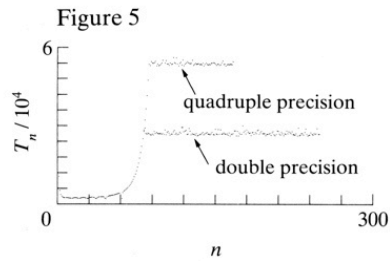
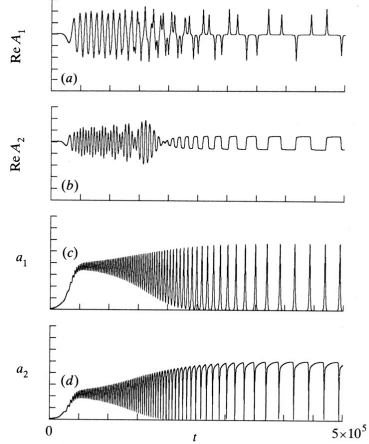
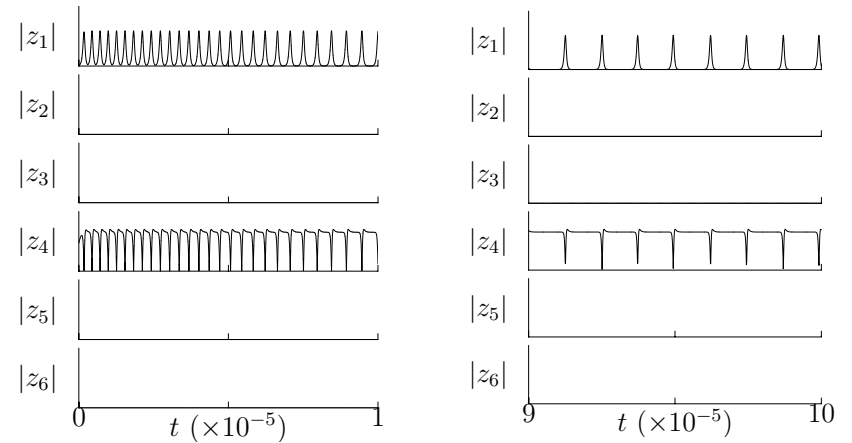


Figure 4. Time interval of the spikes in figure 3a and e. the thin straight line denotes (5.7) derived by Proctor & Jones:  $T_{n-1}/T_n = 1.1548$ . The dashed line denotes  $T_{n+1}/T_n = 1$ . The plotted points for  $T \rightarrow \infty$  are attracted in a heart-shaped area which is almost symmetric about the dashed line.

Figure 5. Evolution of the time interval of the spikes for figure 3a and e. To obtain this figure, numerical integration of (5.1) is carried out over  $t \in [0, 5 \times 10^6]$  with  $4 \times 10^8$  time steps.

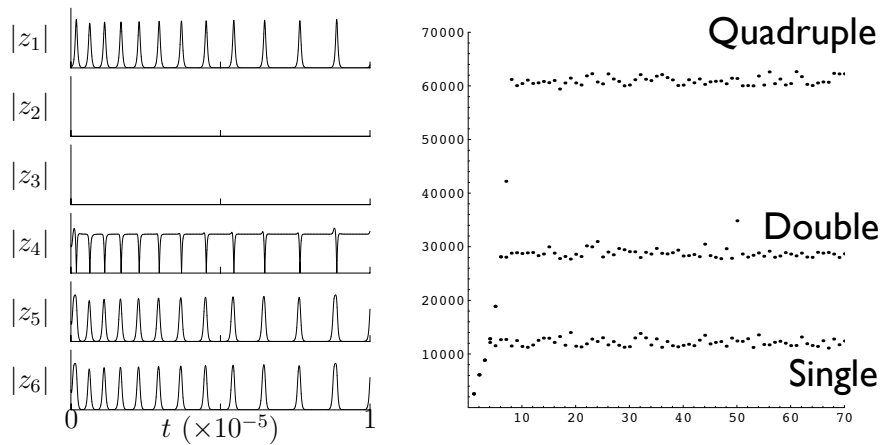
$$(z_1, z_2, z_3, z_4, z_5, z_6) = (10^{-4}, 0, 0, 10^{-3}i, 0, 0)$$



$$I_2 = \mathbf{C}\{(1, 0, 0, 0, 0, 0), (0, 0, 0, 1, 0, 0)\}$$

## 4次元不変部分空間の近ヘテロクリニックサイクル

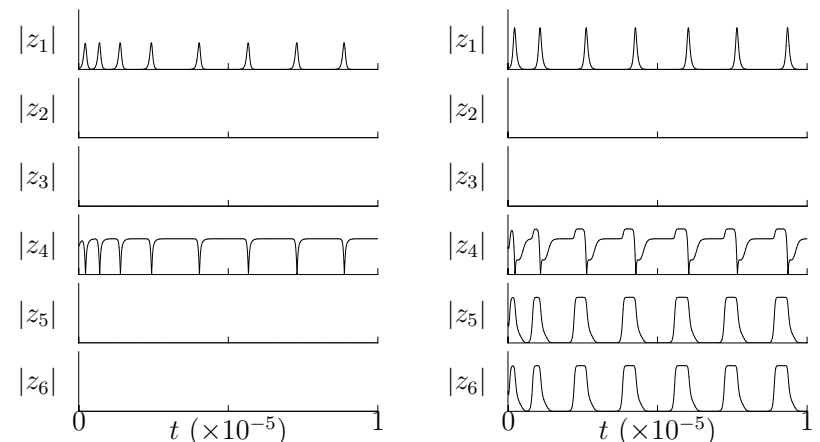
$$P_1 = 150.76, P_2 = 7, \epsilon_1 = 0, \epsilon_2 = 0.1$$



$$I_4 = \mathbf{C}\{(1, 0, 0, 0, 0, 0), (0, 0, 0, 1, 0, 0), (0, 0, 0, 0, 1, 0), (0, 0, 0, 0, 0, 1)\}$$

## 2, 4次元不変部分空間の近ヘテロクリニックサイクル

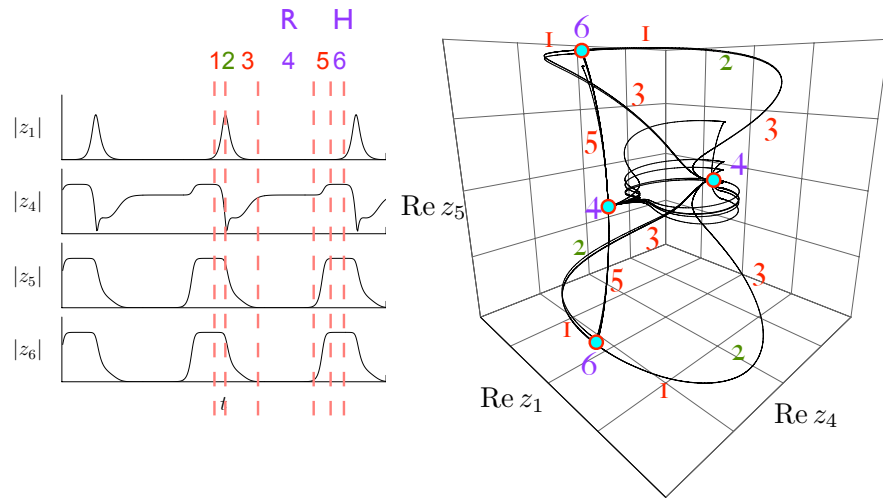
$$I_2 = \mathbf{C}\{(1, 0, 0, 0, 0, 0), (0, 0, 0, 1, 0, 0)\} \quad (10^4, \delta_2, \delta_3, 10^{-3}i, \delta_5, \delta_6)$$



$$\delta_n = 0 \quad \delta_n = O(10^{-24})$$

$$P_1 = 150.76, P_2 = 67, \epsilon_1 = 0, \epsilon_2 = 0.1$$

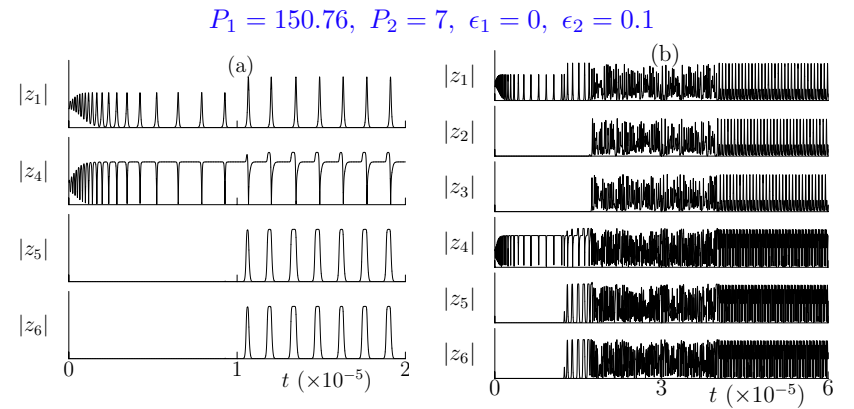
# 4次元不変部分空間の近ヘテロクリニックサイクル



$P_1 = 150.76, P_2 = 7, \epsilon_1 = 0, \epsilon_2 = 0.1$   
 $\rho = 10^{-3}$

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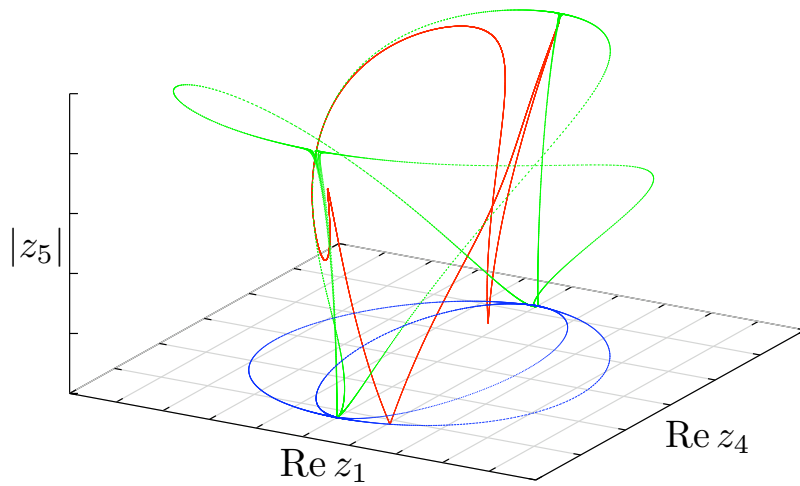
# 近ヘテロクリニックサイクルの崩壊



$I_2 \rightarrow I_4$   
 $\delta_n = O(10^{-24})$

$I_2 \rightarrow I_4 \rightarrow \mathbb{C}^6$   
 $\delta_n = O(10^{-8})$

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$HC(I_2), HC(I_4), \text{ and } OS(\mathbb{C}^6)$

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