



Seismic Response of a Linearly Stable, Misaligned Multirotor System[©]

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Nonlinear behavior of a multirotor system supported by oil film bearings is investigated theoretically by calculating loci of centers of disks and journals of the system using a time marching method. The oil film force of bearings is obtained by solving the Reynolds equation at each time step by a rapid solution presented previously by the authors. It will be shown that a two-disk/four-bearing rotating system operated under the linear stability limit but above twice the first critical becomes unstable when sufficiently large external shocks, an artificial sinusoidal shock and real seismic waves are given to the system. The effect of misalignment of bearings is also examined closely in this study.

KEY WORDS

Seismic Response, Nonlinear, Misalignment, Multirotor System, Numerical Simulation

INTRODUCTION

With the advance of industrial technology, rotary machines like generators, turbines, compressors, pumps, etc.

become large-scale and high-speed. These rotary machines are operated at a higher rotational speed than the first critical speed in many cases. Because rotary machines are operated continuously, oil film bearings used in these machines must maintain their stable condition without breakdown even though external shocks, e.g., seismic waves, work on them. Thus an accurate prediction of vibration characteristics must be made in order to design and manufacture the highly reliable rotary machine. Vibration characteristics of a rotor system and response to external shocks, such as seismic waves, are studied mainly by a linear theory (1), (2), but authors such as Adams (3) and Hori (4) have pointed out the importance of adequately designing rotating machinery to withstand a major earthquake.

From this point of view, the authors investigated theoretically in previous works (5), (6) the stability of a rotor system, Jeffcott rotor and two-disk/four-bearing rotor system, on which artificial sinusoidal waves and seismic waves work, considering nonlinear characteristics of oil film force. They showed that the rotor system becomes unstable and rotors begin to whirl when the rotor system is operated at the speed more than twice the first critical, even if the rotor system is stable in a linear sense and/or even if the rotor system is supported by bearings expected to have high safety, like lemon-shape bearings. Thus the authors pointed out

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NOMENCLATURE

A	= acceleration amplitude of sinusoidal shock given to bearings
a_n	= $l_n/(E_n I_n)$
b_n	= $l_n^2/(2E_n I_n)$
d_n	= $l_n^3/(6E_n I_n)$
E_n	= Young's modulus
F_{xn}, F_{yn}	= in x direction and y direction respectively components of oil film force
G	= gravity acceleration
h	= $1 + \kappa \cos \theta$: nondimensional film thickness
I_n	= second moment of area
I_{Dn}	= diametral moment of inertia
I_{Pn}	= polar moment of inertia
l_n	= length of shaft

L	= width of oil film bearing
m_n	= concentrated mass
p	= pressure
r	= radius of oil film bearing
t	= time
x_n, y_n	= displacement of vertical and horizontal direction
\ddot{x}_B	= acceleration in the x direction given to bearings
ζ	= coordinate of shaft direction
η	= viscosity of lubricating oil
θ	= bearing angle
θ_{xn}, θ_{yn}	= inclination of horizontal and vertical axis
κ	= eccentricity ratio
ω	= rotational speed of shaft
ω_E	= frequency of sinusoidal shock given to bearing
m_a	= misalignment value
n	= indicates nth element.

that linear stability judgement is not adequate when a rotor system is operated under conditions like this. As for the boundary condition used in the calculation, the authors also found that the half-Sommerfeld condition, which is convenient but less exact, gives nearly the same results compared to the quasi-Reynolds condition, which is more exact but requires more calculation time.

Rotor shafts of multirotor systems are usually set up in catenary state, i.e., the mating ends of each neighboring shaft have the same height and the same inclination. One of the authors reported that the misalignment of support bearings from this state has a considerable effect on the linear stability of a multirotor system (7). In this research, the effect of the misalignment on vibration response, especially against a large impulse, of a rotor system is investigated. A sinusoidal shock, 1 period, and seismic waves are given to bearings externally, and vibration response of the rotor system is calculated by the Runge-Kutta-Gill method. The oil film force is obtained by solving the finite width Reynolds equation under the half-Sommerfeld condition at each time step by the solution of the matrix method (8).

THEORY

Modeling of a Rotor System

To obtain the vibration response of a rotor system, the system is approximated by a chain of massless flexible shafts and masses on which a moment of inertia concentrates, shown in Fig. 1.

When an external force acts on the bearings, the equation of motion of the n th element is written as follows:

$$\begin{aligned} m_n(\ddot{x}_n + \ddot{x}_B) - \frac{2}{d_{n-1}} x_{n-1} + 2\left(\frac{1}{d_{n-1}} + \frac{1}{d_n}\right) x_n \\ - \frac{2}{d_n} x_{n+1} - \frac{3}{b_{n-1}} \theta_{xn-1} - 3\left(\frac{1}{b_{n-1}} - \frac{1}{b_n}\right) \theta_{xn} \\ + \frac{3}{b_n} \theta_{xn+1} + F_{xn} - m_n G = 0 \end{aligned} \quad [1]$$

$$\begin{aligned} I_{Dn} \ddot{\theta}_{xn} + I_{Pn} \omega \dot{\theta}_{yn} + \frac{3}{b_{n-1}} x_{n-1} - 3\left(\frac{1}{b_{n-1}} + \frac{1}{b_n}\right) x_n \\ - \frac{3}{b_n} x_{n+1} + \frac{2}{a_{n-1}} \theta_{xn-1} + 4\left(\frac{1}{a_{n-1}} + \frac{1}{a_n}\right) \theta_{xn} \\ + \frac{2}{a_n} \theta_{xn+1} = 0 \end{aligned} \quad [2]$$

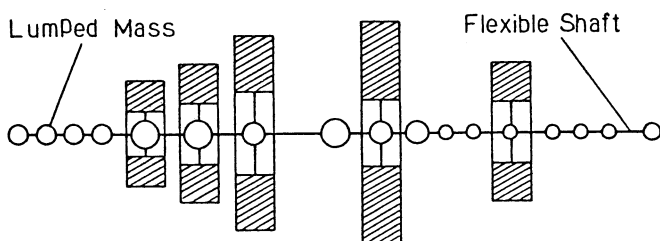


Fig. 1—Model of rotor system.

The equation of motion in the y -direction is omitted here but easily written by replacing x in Eqs. [1] and [2] by y . Thus $4N$ differential equations are obtained when the rotor system is divided into N elements. In this research, the static equilibrium position of the rotating system is obtained by the transfer matrix method, and vibration response is obtained by the Runge-Kutta-Gill method.

Oil Film Force

In the case of infinitesimal vibration of the rotor around the static equilibrium position, the oil film force in the bearing can be approximated by a linear function with respect to the position of the journal center. But when the journal center vibrates in large amplitude due to, e.g., an external shock like an earthquake, the oil film force varies from position to position of the journal in the bearing, thus the oil film force must be estimated numerous times with the motion of the journal. In this study, the Reynolds equation is solved at each time step in the Runge-Kutta-Gill method by the rapid solution developed by the authors (8).

The Reynolds equation for finite width bearings is written as follows:

$$\frac{1}{r^2} \frac{\partial}{\partial \theta} \left(\frac{h^3}{6\eta} \frac{\partial p}{\partial \theta} \right) + \frac{\partial}{\partial \zeta} \left(\frac{h^3}{6\eta} \frac{\partial p}{\partial \zeta} \right) = \omega \frac{\partial h}{\partial \theta} + 2 \frac{\partial h}{\partial t} \quad [3]$$

In the previous work (6), a rotor response was obtained by solving the Reynolds equation under two boundary conditions, half-Sommerfeld condition and quasi-Reynolds condition (8). The simulated results showed that the amplitude of whirling obtained under the former condition was slightly larger, but both results coincided concerning the stability, i.e., the occurrence of whirling motion. Thus throughout the present research, the half-Sommerfeld condition is adopted, as shown in Fig. 2:

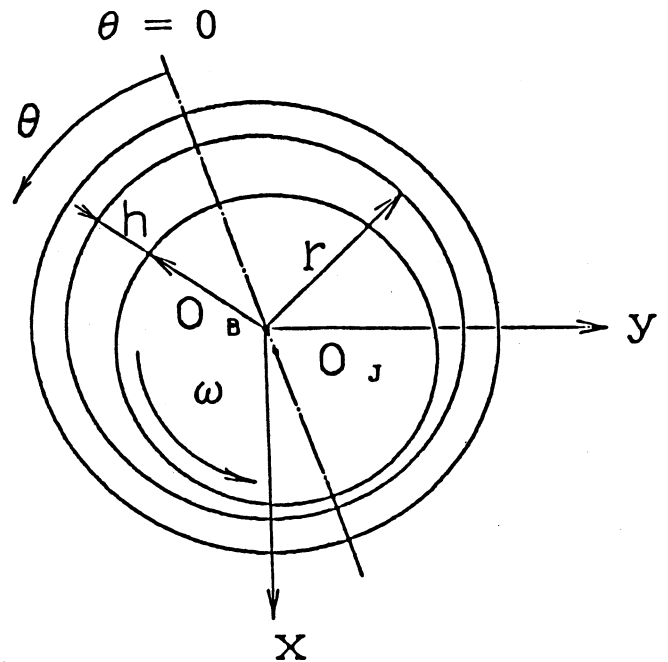


Fig. 2—Coordinates of journal.

$$p = 0 \quad \text{at } \zeta = 0 \quad \text{and} \quad \zeta = L \quad [4]$$

$$p = 0 \quad \text{at } \theta = 0 \quad \text{and} \quad \theta = \pi \quad [5]$$

Misalignment of Support Bearings

In this study, a two-disk/four-bearing multirotor system, two spans, is considered, shown in Fig. 3, and the misalignment of the system is given as follows. A stationary rotor system is assumed and one of the spans is set horizontally with two support bearings, No. 1 and No. 2, fixed at the same height. When the aligned state is required, the bearings of the second shaft, No. 3 and No. 4, are set so that the mating ends of the two shafts have the same height and the same inclination, the catenary-type alignment. The misaligned rotor is created by moving bearings No. 3 and No. 4 simultaneously so that the displacement ratios of No. 3 and No. 4, respectively:

$$\frac{x_3 - x_{30}}{|x_{30}|} \quad \text{and} \quad \frac{x_4 - x_{40}}{|x_{40}|} \quad [6]$$

coincide with each other, namely:

$$\frac{x_3 - x_{30}}{|x_{30}|} = \frac{x_4 - x_{40}}{|x_{40}|} = m_a : \text{misalignment value.} \quad [7]$$

where x_{30} , x_{40} are x -coordinates of bearing No. 3 and No. 4, respectively, in catenary state, and x_3 , x_4 are the x -coordinates of bearing No. 3 and No. 4, respectively, of the misaligned system. For example, the bearings are not misaligned if $m_a = 0$, and all bearings are on a horizontal line if $m_a = +1.0$, shown in Fig. 4.

For the sake of simplicity bearings No. 3 and No. 4 are moved in the x -direction only.

Calculation Method

It is assumed in the simulation that the system operates steadily before an external force acts on it, namely the simulation begins with the static equilibrium state, which is statically indeterminate, thus iterative calculation is needed to obtain the initial condition. Then the Runge-Kutta-Gill method is used to calculate the response of the rotor system to external force. Time step width Δt adopted in the calculation is 0.05 ms and unknown variables for the n th element are \dot{x}_n , x_n , $\dot{\theta}_{xn}$, θ_{xn} , \dot{y}_n , y_n , $\dot{\theta}_{yn}$, θ_{yn} .

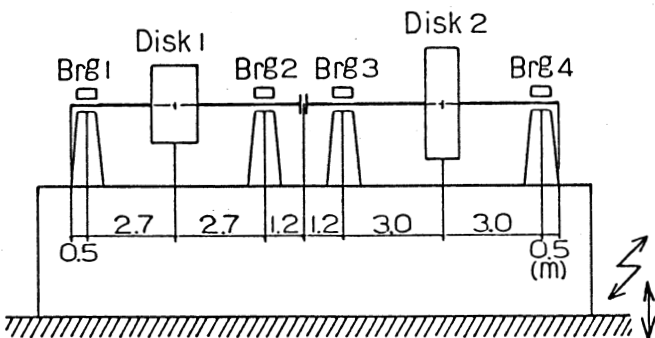


Fig. 3—Calculation model.

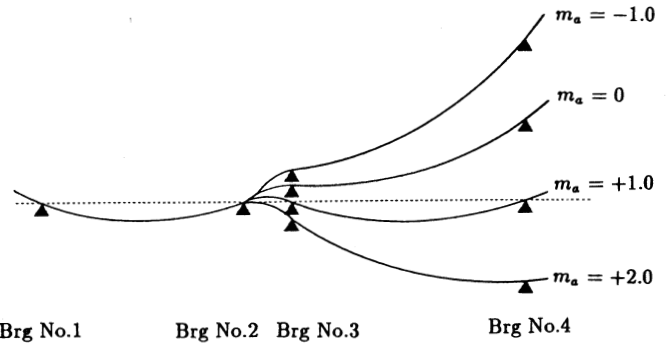


Fig. 4—Misalignment.

CALCULATION MODEL

The two-disk/four-bearing rotor system shown in Fig. 3 is adopted as a model rotor. The details of the rotor and bearings are described in Table 1 and Table 2, respectively. When bearings are aligned, $m_a = 0$, the natural frequency of the first bending mode of the model rotor system is about 400 rpm and the linear stability limit is about 4500 rpm. Thus, the rotational speed, 3600 rpm, adopted in the calculation is higher than twice the first critical but lower than the linear stability limit.

CALCULATION RESULTS

Response to Sinusoidal Shock

The response of the rotor system to an external sinusoidal shock, 1 period, in the lateral direction is calculated. The shock is given by:

$$\ddot{X}_B = 0, \quad \ddot{Y}_B = \begin{cases} A \sin(\omega_E t) & 0 \leq t \leq 2\pi/\omega_E \\ 0 & 2\pi/\omega_E \leq t \end{cases} \quad [8]$$

where ω_E is the frequency of the shock. The natural frequency of the system is used for this value because the rotor begins to swirl easily when ω_E is close to the natural frequency, as shown in the previous study (5) by the authors. The amplitude A used in the calculation was 0.15 G and 40000 steps are calculated in the simulation. The dynamic responses of bearing No. 2 are shown in Figs. 5(a)–5(g), where

TABLE 1—SPECIFICATION OF ROTOR SYSTEM	
Total Mass of Rotor System	8.1×10^4 Kg
Mass of Disk No. 1	2.7×10^4 Kg
Mass of Disk No. 2	4.1×10^4 Kg
Total length of Rotor System	14.8 m
Distance from Bearing No. 1 to No. 2	5.4 m
Distance from Bearing No. 2 to No. 3	2.4 m
Distance from Bearing No. 3 to No. 4	6.0 m
Elastic Modulus of Shaft	2.1×10^{11} N/m ²

TABLE 2—SPECIFICATION OF BEARING	
Radius of Bearing	0.6 m
Width of Bearing	0.3 m
Radial Clearance	0.9 mm
Viscosity of Lubricant	6.0 mPa·s

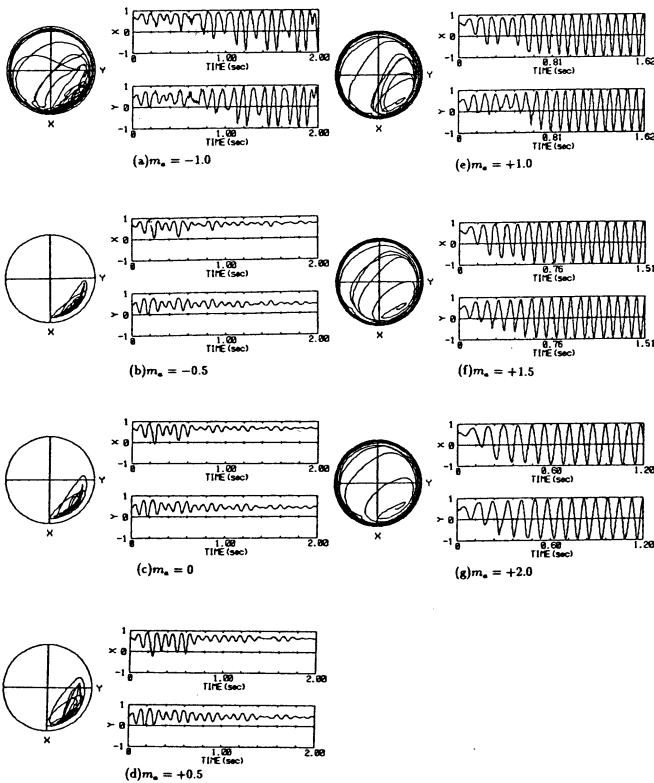


Fig. 5—Response to sinusoidal shock, Bearing No. 2.

misalignment m_a is varied from -1.0 to $+2.0$. The left circle represents the locus of the journal center and the right diagram graphs the displacement of the journal center in the x -direction (vertical) and y -direction (horizontal). Only the results of bearing No. 2 are shown in the figures because this bearing tends to become unstable more easily than the others.

It is seen from these figures that the nonlinear stability of a rotor system is affected by the misalignment, but the linear and nonlinear stabilities do not always coincide with each other. For example, Fig. 5 shows that the system has high resistance to the external shock when $m_a = -0.5$, $m_a = 0$, and $m_a = +0.5$, compared to the other misalignment, which does not coincide with the linear stability, calculated and shown in Fig. 6. Accordingly, when an external shock is expected to work on a rotor system which is operating at the speed more than twice the first critical, and in particular supported by misaligned bearings, linear stability theory alone is not adequate for the prediction of the rotor dynamic characteristics. The nonlinear theory as presented in this study must be used for this purpose.

Misalignment for Suppressing Whirling

The authors found in the previous section that misalignment has a considerable effect on the nonlinear stability of the rotor system. Thus it is considered that the dynamic characteristics of a rotor system can be improved by the misalignment of the bearings. Figs. 7 and 8 show the responses when the misalignments are $m_a = 0$ and $m_a = -0.5$, respectively, which are cases of relatively smaller whirling motions compared to other misalignments. It is also seen from these figures that the amplitude of vibration of bearing

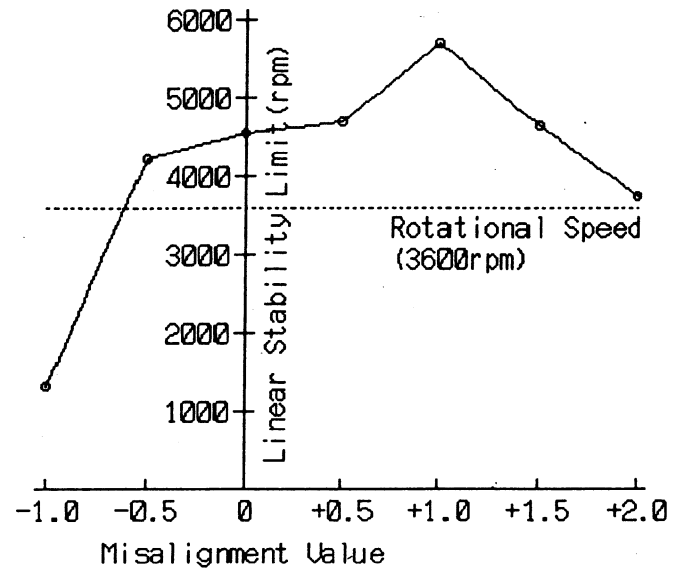
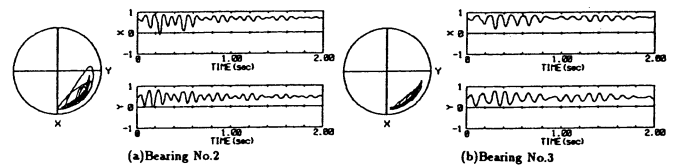
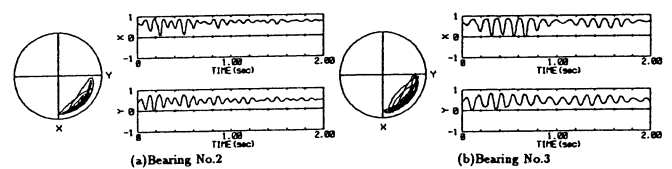


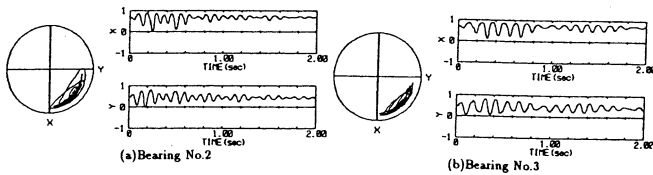
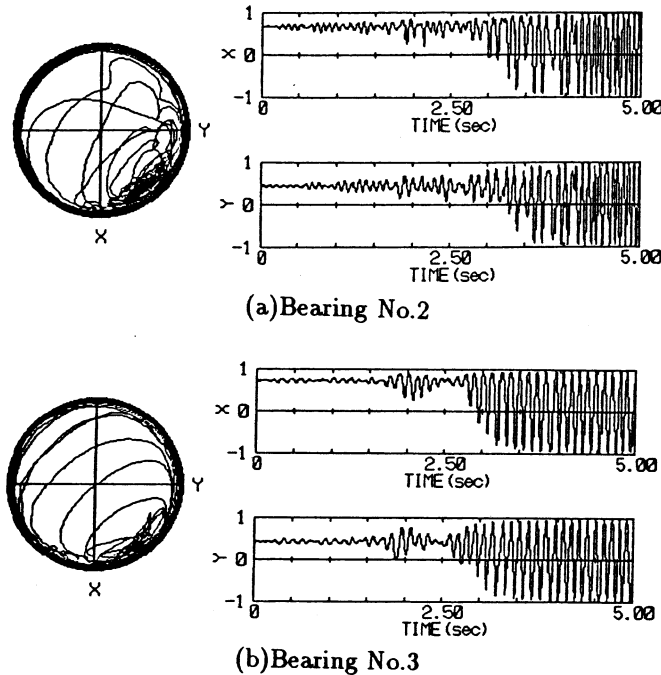
Fig. 6—Misalignment value and linear stability limit.

Fig. 7—Response to sinusoidal shock, $m_a = 0$.Fig. 8—Response to sinusoidal shock, $m_a = -0.5$.

No. 2 is larger than that of No. 3 in the case of $m_a = 0$, Fig. 7, but that of bearing No. 3 is larger than No. 2 in case of $m_a = 0.5$, Fig. 8. Thus it is expected that the dynamic characteristics can be improved by considering the case of $m_a = -0.25$. The results of this case are shown in Fig. 9. It is seen from this figure that the whirling motion of both bearings No. 2 and No. 3 becomes smaller than those in Fig. 7 and Fig. 8.

Response to Real Seismic Wave

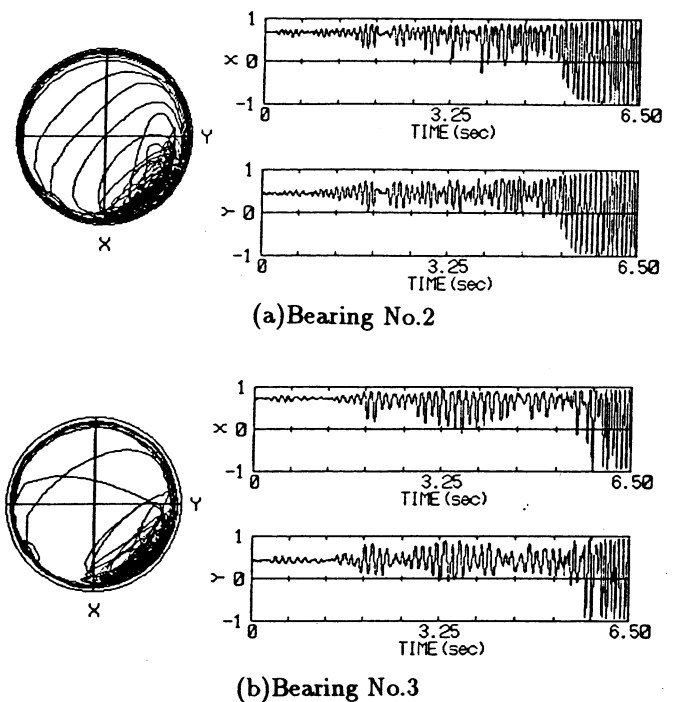
Acceleration in the east-west direction and the vertical direction of a real seismic wave (5) (1952, Taft, U.S.A) is given to the rotor system. Maximum acceleration of this earthquake is about $0.18 G$ in the east-west direction and about $0.1 G$ in the vertical direction. From the FFT analysis of these seismic waves, it was found that components of frequency less than 10 Hz were dominant (5). The time step used in the Runge-Kutta-Gill method Δt is 0.05 ms . Figure 10 and Fig. 11 show the calculation result for the cases that $m_a = 0$ and $m_a = -0.25$, respectively. It is seen from these figures that the rotor begins to whirl about three seconds

Fig. 9—Response to sinusoidal shock, $m_a = -0.25$.Fig. 10—Response to seismic wave, $m_a = 0$.

after the earthquake occurred in the case of $m_a = 0$, but about five seconds in the case of $m_a = -0.25$. Thus $m_a = -0.25$ has higher resistance to the earthquake than $m_a = 0$.

CONCLUSION

The dynamic responses of a two-disk/four-bearing multirotor system to an artificial sinusoidal shock and to real seismic waves are investigated considering nonlinear characteristics of the oil film force and the misalignment of support bearings. The effect of misalignment of bearings on vibration response of the rotor system is considerable, and the response is different from the expectation obtained from the linear stability analysis concerning the model rotor adopted in the present study. Therefore, the linear theory is not adequate, and the nonlinear calculation such as used

Fig. 11—Response to seismic wave, $m_a = -0.25$.

in this research is needed to design safety rotor systems which are protected against a large shock such as earthquake.

It is also shown that high stability in a nonlinear sense can be achieved by varying slightly the misalignment of the support bearings.

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