INSTABILITY ANALYSIS OF ULTRA-THIN POLAR LUBRICANT SURFACE

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Introduction
To examine behaviors and stabilities of ultra-thin but continuum liquid films, the long wave theory [1] was employed, which is the time-evolution equation for the shape of the thin liquid film and includes surface tensions and surface forces such as the van der Waals (vdW) forces. For lubricants with chain-end functional groups such as Z dol, surface forces acting at the liquid surface depend on the liquid thickness, and the direction of the force changes alternatively with the thickness. In this report stability/instability of the polar lubricant surfaces was analyzed as the growth and decay problem of infinitesimal and sinusoidal disturbances of the liquid surface.

Long Wave Equation for Gas-liquid Interface
The simplest form of the long wave equation for film thickness \( h_L \) is expressed by the following equation:

\[
\frac{\partial h_L}{\partial t} + \frac{1}{3\mu_L} \frac{\partial}{\partial x} \left( h_L^3 \frac{\partial}{\partial x} \left( -\Pi - \gamma_{OL} \frac{\partial^2 h_L}{\partial x^2} \right) \right) = 0, \quad (1)
\]

where \( \gamma_{OL} \) is the surface tension and \( \Pi \) is the van der Waals pressure.

This equation can give not only microscopic flow and deformation of the liquid surface, but also stability/instability discrimination [2].

Linear Stability Analysis of Liquid Surface
Assuming infinitesimal and sinusoidal disturbances of the liquid surface and linearizing Eq. (1), the time-evolution of the liquid film, \( h_L \), can be examined. The infinitesimal disturbances of the liquid film is expressed as

\[
h_L = h_{L0} \left( 1 + \varepsilon \exp(ikx + st) \right), \quad (2)
\]

where \( h_{L0} \) is the liquid film thickness of the stationary state, \( k \) is the wave number of sinusoidal waves and \( \varepsilon << 1 \). The coefficient \( s \) is the growth rate of the disturbance, and for \( \text{Re}[s] < 0 \) disturbances at the surface decay.

van der Waals Pressure at the Polar Liquid Surface
The van der Waals pressure \( \Pi \) acting at lubricant surfaces of polar liquids on the recording disk is composed of the dispersion term \( p_{dis} \) and the polar term \( p_{polar} \):

\[
\begin{align*}
\text{Gas: } & \quad n = 1.0, \\
\text{Liquid: } & \quad n = 1.3, \\
\text{Solid: } & \quad n = 1.9
\end{align*}
\]

![Fig. 1 van der Waals pressure at thin liquid film on solid](image)

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\[ \Pi = p_{\text{vis}} + p_{\text{polar}}, \quad (3) \]

where
\[ p_{\text{vis}} = -A_{12}/6\pi h_L^3, \quad (4) \]
\[ p_{\text{polar}} = \left( \frac{S^p}{l_n} - \frac{n_{12}^2 \varepsilon_{12}^p}{h_n} \sin \left( \frac{n_{12}^p}{h_n} + \alpha \right) - \frac{\Delta \varepsilon_{12}^p}{l_n} \cos \left( \frac{n_{12}^p}{h_n} \right) \right) \frac{l_n^2}{h_n} \epsilon^2. \quad (5) \]

Figure 2 shows pressures \( \Pi, p_{\text{vis}} \) and \( p_{\text{polar}} \). As the polar term \( p_{\text{polar}} \) is repulsive and attractive to the solid depending on the film thickness \( h_L \), the total vDW pressure \( \Pi \) has minimum and maximum values for thicknesses of \( h_1, h_2, h_3 \), etc. For refractive indices of \( n_1=1.9, (DLC) \), \( n_2=1.3, (PFPE) \) and \( n_3=1, (Air) \), the Hamaker constant \( A_{12} = -4.7 \times 10^{-20} \text{J} \). Constants in Eq. (5) are the same as those in Refs. [3] and [4].

**Stability/Instability in Polar Liquid Surface**

Relationship between the growth rate \( s \) and the wave number \( k \) is
\[ s = (h_L^3 k^2 / 3 \mu_L) \left[ f(h_L) - \gamma_{12} h_L k^2 \right], \quad (6) \]
where \( f(h_L) \) is the quantity which depends only on the thickness in the stationary state, \( h_{L0} \), and corresponds to the derivative \( d\Pi / dh_L \).

Figure 3 (a) is the relationship between dimensionless growth rate \( S \) and dimensionless wave number \( K \). For positive \( S \), the liquid surface is unstable. As the wave number \( K \) increases, the surface becomes stable because stabilizing effects of surface tensions exceed to the instability. The black circles indicate the critical states in stability/instability and the white circles show the most unstable states.

Figure 3 (b) shows the critical wave number \( k_c \) and the most unstable wave number \( k_{\text{max}} \):
\[ k_c = \sqrt{f(h_L)/\gamma_{12} h_L^3}, \quad (7) \]
\[ k_{\text{max}} = \sqrt{f(h_L)/2\gamma_{12} h_L} = k_c / \sqrt{2}. \quad (8) \]

The thicknesses \( h_1, h_2, h_3 \), etc. in Fig. 3 (b) correspond to those in Fig. 2.

**References**